Continuous Cocurrent Processes in the Nonsteady State

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Although continuous heat- and mass-transfer processes are usually conducted in countercurrent flow, there are certain cases in which the cocurrent flow can be used to some advantages from an operation point of view. For example, in the packed adsorption column, flooding due to high vapor or gas flow rate can be eliminated if cocurrent flow is employed. Use of cocurrent flow can also reduce the pressure drop requirement in the packed column. Reiss (1967) discussed several instances in which cocurrent mode operation may be favored over countercurrent flow. In the case of heat transfer, cocurrent flow is also suitable for some special situations. For instance, if it is necessary to limit the maximum temperature of cooler fluid or if it is important to change the temperature of at least one fluid rapidly (McCabe et al., 1985). The transient behaviors of both cocurrent and countercurrent processes are important in the startup and control of mass- and heat-transfer operations. Previously, numerical solutions based on a method of characteristics was used to study the transients of countercurrent processes (Tan and Spinner, 1984). Owing to the split boundary conditions, analytic solutions are difficult to obtain even for the linear system. For cocurrent flow, compact analytic solutions in terms of well-known tabulated function can be derived. The purpose of this note is to present the nonsteady-state behavior of continuous cocurrent flow of two contacting phases. The transient responses to the inlet disturbances are derived based on a simple mathematical model. This simplified model is equally applicable to either mass- or heat-transfer processes. Laplace transformation method is used to obtain compact analytic solutions. These solutions can be easily evaluated with the aid of the known tabulated mathematical functions or charts. Both the derivation and the final form of solutions are easier to apply in comparison to the work of Li (1986).

Model Equations and Solutions

Plug flow, constant physical properties and no axial dispersion are assumed. Linear driving force rate equations are used for both the mass- and heat-transfer processes. For mass transfer, the transfer species is assumed to be dilute compared to the carrier component. Thus, constant molar flow and linear isotherm assumptions are expected to be valid. The model equations for mass transfer are:

$$G_x \frac{\partial x}{\partial z} + \beta_x \frac{\partial x}{\partial t} = K_y a S (y - y^*) \tag{1}$$

$$G_y \frac{\partial y}{\partial z} + \beta_y \frac{\partial y}{\partial t} = K_y a S (y^* - y) \tag{2}$$

and for heat transfer

$$C_{p1} G_1 \frac{\partial T_1}{\partial z} + \beta_1 C_{p1} \frac{\partial T_1}{\partial t} = UaS (T_2 - T_1) \tag{3}$$

$$C_{p2} G_2 \frac{\partial T_2}{\partial z} + \beta_2 C_{p2} \frac{\partial T_2}{\partial t} = UaS (T_1 - T_2) \tag{4}$$

In Eqs. 1 and 2, with the linear isotherm assumption, $y^* = mx$, where m is the slope of the linear isotherm.

Equations 1 and 2 and Eqs. 3 and 4 can be transformed to the following identical form of dimensionless equations:

$$\frac{\partial X}{\partial h} + \frac{\partial X}{\partial \theta} = Y - X \tag{5}$$

$$A \frac{\partial Y}{\partial h} + B \frac{\partial Y}{\partial \theta} = X - Y \tag{6}$$

where the normalized variables and parameters are defined by:

	mass transfer	heat transfer
X	mx	$(T_2 - T_{2o})/(T_{1o} - T_{2o})$
Y	y	$(T_1 - T_{2o})/(T_{1o} - T_{2o})$
h	$\frac{mK_y aSz}{G_x}$	$\frac{UaSz}{G_2 C_{p2}}$
θ	$\frac{mK_y aSt}{\beta_x}$	$\frac{UaSt}{\beta_2 C_{p2}}$
A	$\frac{mG_y}{G_x}$	$\frac{C_{p1} G_1}{C_{p2} G_2}$
B	$\frac{m\beta_y}{\beta_x}$	$\frac{C_{p1}\beta_1}{C_{p2}\beta_2}$

To solve Eqs. 5 and 6, we assume the following initial conditions $X(h,0)=X^*(h)$ and $Y(h,0)=Y^*(h)$ where $X^*(h)$ and $Y^*(h)$ are the initial steady-state values given by:

$$X^*(h)=\frac{1}{A+1}[X_o^*+AY_o^*]+\frac{A}{A+1}[X_o^*-Y_o^*]\exp\left[-\frac{(A+1)h}{A}\right] \quad (7)$$

$$Y^*(h)=\frac{1}{A+1}[X_o^*+AY_o^*]-\frac{1}{A+1}[X_o^*-Y_o^*]\exp\left[-\frac{(A+1)h}{A}\right] \quad (8)$$

For the boundary conditions we consider the step input changes in which inlets of X and Y are suddenly changed from X_o^* to X_o and Y_o^* to Y_o:

$$X(0,\theta>0)=X_o \quad \text{and} \quad Y(0,\theta>0)=Y_o$$

The initial conditions as stated imply that prior to the new solution front of the faster moving stream reaching the axial distance z, X and Y values will remain at initial steady state. Thus, if we assume that after the input change, the new solution front of Y advances ahead of new solution front of X (that is, $B/A<1$) and then the initial conditions can also be expressed as:

$$X(h,\theta<Bh/A)=X^*(h) \quad \text{and} \quad Y(h,\theta<Bh/A)=Y^*(h)$$

The quantity $\theta-Bh/A$ thus represents the contact time for Y stream at a given location of z. Define $\theta'=(\theta-Bh/A)/(1-B/A)$ and by expressing X and Y in terms of deviation from the initial steady state, the following equations are derived from Eqs. 5 and 6:

$$\frac{\partial\hat{X}}{\partial h}+\frac{\partial\hat{X}}{\partial\theta'}=\hat{Y}-\hat{X} \quad (9)$$

$$A\frac{\partial\hat{Y}}{\partial h}=\hat{X}-\hat{Y} \quad (10)$$

where $\hat{X}(h,\theta')=X(h,\theta')-X^*(h)$, $\hat{Y}(h,\theta')=Y(h,\theta')-Y^*(h)$ with boundary conditions given by:

$$\hat{X}(0,\theta'>0)=\hat{X}_o=X_o-X_o^*$$

$$\hat{Y}(0,\theta'>0)=\hat{Y}_o=Y_o-Y_o^*$$

By Laplace transformation of Eqs. 9 and 10 it can be shown that:

$$\hat{X}=\frac{\hat{X}_o(Ap+1)+A\hat{Y}_o}{s[(Ap+1)(p+s+1)-1]} \quad (11)$$

$$\hat{Y}=\frac{A\hat{Y}_o(p+s+1)+\hat{X}_o}{s[(Ap+1)(p+s+1)-1]} \quad (12)$$

where s is the transform variable with respect to θ' and p is the transform variable with respect to h. The inversions of the above equations are:

$$\hat{X}=\frac{1}{A+1}[\hat{X}_o+A\hat{Y}_o]+\frac{A}{A+1}[\hat{X}_o-\hat{Y}_o]\exp\left[-\frac{(A+1)h}{A}\right]+g(h-\theta')\frac{A}{A+1}[\hat{X}_o-\hat{Y}_o]\left[1-J\left(\frac{h-\theta'}{A},\theta'\right)\right]-\hat{X}_oJ\left(\theta',\frac{h-\theta'}{A}\right)+g(h-\theta')\frac{A}{A+1}[\hat{X}_o-\hat{Y}_o]\exp\times\left[\frac{A-1}{A}(h-2\theta')\right]\psi\left(h-\theta',\frac{\theta'}{A}\right) \quad (13)$$

$$\hat{Y}=\frac{1}{A+1}[\hat{X}_o+A\hat{Y}_o]-\frac{1}{A+1}[\hat{X}_o-\hat{Y}_o]\exp\left[-\frac{(A+1)h}{A}\right]-g(h-\theta')\frac{1}{A+1}[\hat{X}_o+A\hat{Y}_o]\left[1-J\left(\frac{h-\theta'}{A},\theta'\right)\right]-g(h-\theta')\frac{1}{A+1}[\hat{X}_o-\hat{Y}_o]\exp\times\left[\frac{A-1}{A}(h-2\theta')\right]\psi\left(h-\theta',\frac{\theta'}{A}\right) \quad (14)$$

The Heaviside function, g, is defined such that $g(h-\theta')=1$ if $h\geq\theta'$ and $g(h-\theta')=0$ if $h<\theta'$. Both solutions for $\hat{X}$ and $\hat{Y}$ contain two mathematical functions. One of these, the J function, is available in tabulated form and on charts (Sherwood et al., 1975; Hougen and Watson, 1947). The other mathematical function, ψ, is derived from the convolution integral in which

$$\psi(u,v)=\exp(-2u)\int_o^u\exp(\lambda-v)I_o(2\sqrt{v\lambda})d\lambda$$

where I_o is the modified Bessel function of the first kind of order zero. The ψ function was used in the analysis of transients of a shell and tube heat exchanger (Tan and Spinner, 1978). Both J and ψ function can be evaluated by the following series form:

$$J(u,v)=1-\sum_o^\infty\frac{v^kA_k(u)}{k!k!}$$

where

$$A_o=1-\exp(-u),\ A_k=kA_{k-1}-u^k\exp(-u),\ \text{for } k\geq1$$

$$\psi(u,v)=\exp(-u-v)\sum_o^\infty\frac{v^kB_k(u)}{k!k!}$$

where

$$B_o=1-\exp(-u),\ B_k=u^k-kB_{k-1},\ \text{for } k\geq1$$

The first two terms on righthand side of both Eqs. 13 and 14 represent the deviation of the final steady state from the initial steady state. They are exactly identical in form to Eqs. 7 and 8 with X_o^* replaced by $\hat{X}_o$ and Y_o^* replaced by $\hat{Y}_o$.

The transient response of $\hat{X}$ and $\hat{Y}$ in Eqs. 13 and 14 can also be normalized in terms of deviation between final steady state and initial steady state. Thus, $\hat{X}/\hat{X}^*(h)$ and $\hat{Y}/\hat{Y}^*(h)$ represent the fractional attainment of final steady state for X and Y respectively, where $\hat{X}^*(h)$ and $\hat{Y}^*(h)$ denote the difference between the two steady-state values.

From the properties of J and ψ functions, it is known that $J(0, v) = 1$ and $\psi(0, v) = 0$, thus when $\theta' \geq h$, the final steady state is reached for Y stream, as can be seen from Eq. 14. The presence of two Heaviside functions in Eqs. 13 and 14 suggests that there are three regions which separate the initial, transient and the final steady state of both X and Y streams. Figure 1 is a sketch on the time-distance diagram which indicates the location of these three regions. For a step input change in either X and Y inlets, a concentration or temperature jump (discontinuity) for the faster moving stream is expected at $\theta' = 0$ for a given location of h. Discontinuity will also occur at $\theta' = 1$, if there is a step change in the slower moving stream.

A special case to the solutions of Eqs. 13 and 14 is when $A = B$. This means that both X and Y solution front will advance at the same speed and both streams have identical contact time. For this special case, the steady state is immediately reached after the elapse of one residence time. Thus, for $\theta' \geq h$, both X and Y assume the steady-state solution in the form of Eqs. 7 and 8 By means of linear superposition and application of Duhamel formula, Eqs. 13 and 14 can also be used to derive the transient responses for time-dependent input changes.

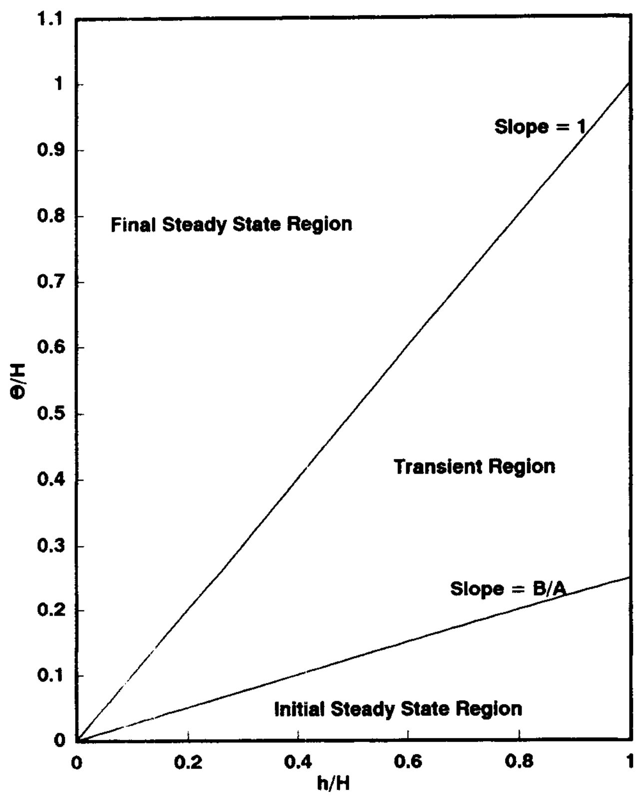

Figure 1. Position of regions for step input changes.

Numerical Examples

Example 1. For a cocurrent gas-liquid contacting system in a packed column, the transient responses of the exit X and Y to a step input change in Y stream are to be determined from the following data: $X_o^* = 0$; $Y_o^* = 1\%$; $X_o = 0$; $Y_o = 2\%$; $m = 1$; $G_x = 2{,}000$ kmol/h; $G_y = 2{,}000$ kmol/h; $\beta_x = 12$ kmol/m; $\beta_y = 1$ kmol/m; $L = 5$ m; $S = 2$ m^2; $K_y a = 400$ kmol/m^3·h.

The calculated dimensionless parameters are $A = 1$, $B = 1/12$ and $H = 2$, where H is the total length parameter corresponding to the value of h at $z = L$.

By setting $h = H$ and applying Eqs. 13 and 14, the exit X and Y in terms of deviation variables, $\hat{X}(H, \theta')$ and $\hat{Y}(H, \theta')$, are calculated. Knowing the initial steady-state value of $X^*(H)$ and $Y^*(H)$ and the relation between θ' and real time, the results obtained are plotted in terms of mole % of X and Y against absolute time, as shown in Figure 2. The evaluation of Eqs. 13 and 14 is facilitated by the use of the available tables and charts.

It is interesting to note that for the step input change in cocurrent flow, the time required to reach the final steady state is solely depending on the residence time of X and Y streams. In this case, the one residence time for X stream is 108 s and for Y stream is 9 s. Due to the step input change in Y inlet, there is a concentration jump at the outlet of the column after one residence time for Y stream. This discontinuity can be verified by substituting $\theta' = 0$ in Eq. 14. When the step input change is introduced, the exit X and Y streams remain at initial steady state prior to the elapsed of one residence time for Y stream. The final new steady-state conditions are realized after the completion of one residence time for X stream. In general, the transient period is shorter for cocurrent flow than in the case for countercurrent flow.

Example 2. A cocurrent flow heat-transfer problem with simultaneous step input changes in both T_1 and T_2 streams is illustrated in this example. The following are the data employed for the calculation: $T_{1o}^* = 35°C$; $T_{2o}^* = 10°C$; $T_{1o} = 60°C$; $T_{2o} = 20°C$; $L = 15$ m; $G_2 = 0.3$ kg/s; $G_1 = 0.6$ kg/s; $C_{p1} = C_{p2} = 4{,}200$ J/kg·K; $U = 600$ J/m^2·s·K; $aS = 0.21$ m^2/m; $\beta_1 = \beta_2 = 2$ kg/m.

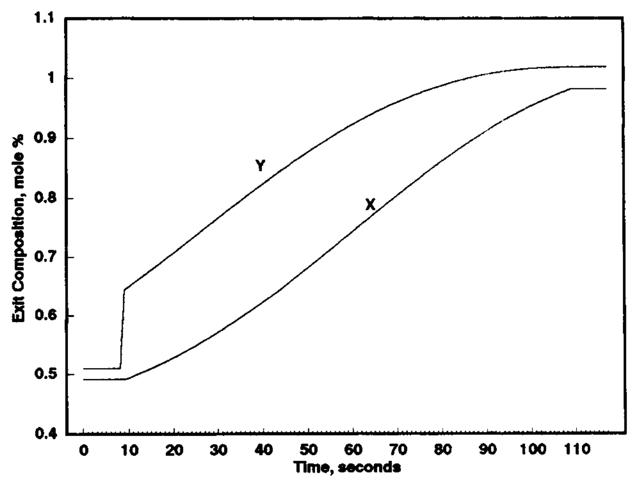

Figure 2. Exit X and Y composition response to a single step input change.

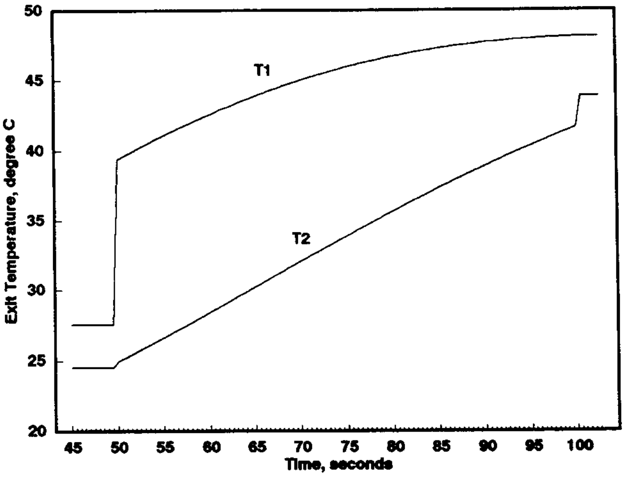

Figure 3. Exit temperature response to simultaneous step input changes.

Values of the normalized parameters are therefore $A=2$, $B=1$ and $H=1.5$.

The residence time for T_1 stream is $\beta_1 L/G_1 = 50$ s while the residence time for T_2 stream is $\beta_2 L/G_2 = 100$ s.

Transient responses of exit temperatures are shown in Figure 3. Note that the time scale is in terms of real absolute time, that is, the time after the step input changes are applied. Because of the sudden step input change in the T_1 stream, there is a discontinuity when it emerges from the outlet after one residence time. Similarly, because of sudden step input change in T_2 there is also a discontinuity for this stream when it emerges from the outlet after completion of one corresponding residence time.

Conclusions

Although continuous countercurrent mass- or heat-transfer processes yield better efficiency there are certain situations which require cocurrent mode of operation. These situations may be due to operational difficulties or to the restriction on the conditions of the process variables. The understanding of transient behavior and startup operation is important for both countercurrent and cocurrent processes. In this note, we have presented an exact and compact analytic solution for cocurrent processes based on a linear mathematical model. The derived solutions are easy to evaluate with the use of available tabulated functions and charts. The analytic solutions obtained also provide an understanding on the dynamic behavior of the cocurrent processes.

Notation

a = interfacial area per unit volume of the contacting equipment, m^2/m^3
A = dimensionless parameter for mass-flow rate ratio
B = dimensionless parameter for holdup ratio
C_p = heat capacity, J/kg-K
C_{p1} = heat capacity of stream 1 fluid
C_{p2} = heat capacity of stream 2 fluid
G = mass- or molar-flow rate, kg/hr or kmol/hr
G_1 = mass-flow rate of T_1 stream
G_2 = mass-flow rate of T_2 stream
G_x = molar-flow rate of x stream
G_y = molar-flow rate of y stream
h = dimensionless axial distance variable
H = dimensionless length parameter
J = mathematical function, dimensionless
K = overall mass-transfer coefficient
K_y = overall mass-transfer coefficient for the lighter phase
L = total length of the contacting equipment, m
m = slope of the linear isotherm, dimensionless
S = cross-sectional area of the contacting equipment, m^2
t = absolute time, s
T = temperature, °C
T_1 = temperature of stream 1 fluid
T_2 = temperature of stream 2 fluid
U = heat-transfer coefficient, $J/m^2 \cdot s \cdot K$
x = mole fraction of the transferring species in the heavy phase
X = normalized composition or temperature variable
y = mole fraction of the transferring species in the light phase
Y = normalized composition of temperature variable
z = axial distance of the contacting equipment, m

Greek letters

β = mass or molar holdup, kg/m or kmol/hr
β_1 = mass holdup of T_1 stream per unit length of exchanger
β_2 = mass holdup of T_2 stream per unit length of exchanger
β_x = molar holdup of x stream per unit length of column
β_y = molar holdup of y stream per unit length of column
ψ = mathematical function, dimensionless
θ = dimensionless time variable

Superscripts

$*$ = steady state, also for equilibrium value (as in y^*)
$\hat{}$ = deviation variable

Subscripts

x = heavy phase stream
y = light phase stream
1 = stream 1
2 = stream 2
o = inlet condition

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